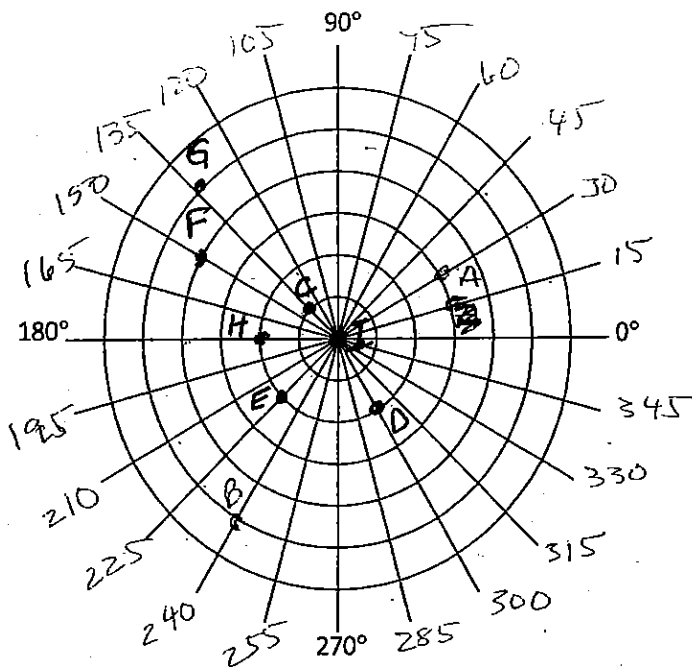


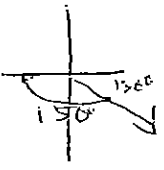
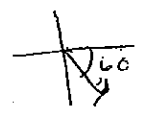
Polar Coordinates

I. Graph and label each point.

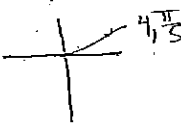
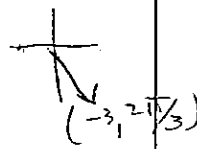
1. $A(3, 30^\circ)$
2. $B(5, 240^\circ)$
3. $C(1, 135^\circ)$
4. $D(2, -60^\circ)$
5. $E(-2, 45^\circ)$
6. $F(-4, 300^\circ)$
7. $G(-5, -45^\circ)$
8. $H(-2, 0^\circ)$
9. $I(0, -270^\circ)$



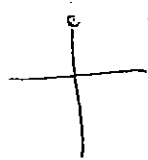
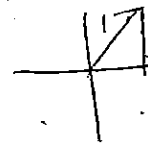
II. State three other pairs of polar coordinates for each point where $-360^\circ < \theta < 360^\circ$. Show work.

<p>10. $(-2, 150^\circ)$ $(-2, -210^\circ)$ $(2, -30^\circ)$ $(2, 330^\circ)$</p> 	<p>11. $(5, -60^\circ)$ $(5, 300^\circ)$ $(-5, 120^\circ)$ $(-5, 240^\circ)$</p> 
--	---

III. State three other pairs of polar coordinates for each point where $-2\pi < \theta < 2\pi$. Show work.

<p>12. $(4, \frac{\pi}{5})$ $(4, -\frac{9\pi}{5})$ $(-4, -\frac{4\pi}{5})$ $(-4, \frac{6\pi}{5})$</p> 	<p>13. $(-3, \frac{2\pi}{3})$ $(-3, -\frac{4\pi}{3})$ $(3, \frac{5\pi}{3})$ $(3, -\frac{\pi}{3})$</p> 
--	--

IV. A point in polar coordinates is given. Convert the point to rectangular coordinates. Show work.

<p>14. $(3, \frac{\pi}{2})$ $(0, 3)$</p> 	<p>15. $(-1, \frac{5\pi}{4})$ $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$</p> 
---	---

16. $(2, \frac{7\pi}{6})$
 $(-\sqrt{3}, -1)$

17. $(-2.5, 1.1)$
 Use a calculator.
 $(-1.13, -2.23)$

$2.5 \sin \theta = 1.1 \Rightarrow \theta = \arcsin\left(\frac{1.1}{2.5}\right)$

V. A point in rectangular coordinates is given. Convert the point to polar coordinates. Show work.

18. $(-3, -3)$
 $(-3\sqrt{2}, \frac{\pi}{4})$

19. $(-6, 0)$
 $(6, \pi)$

20. $(4, -4\sqrt{3})$
 $(8, -\frac{\pi}{3})$

21. $(-3, 4)$
 $(5, \arctan(\frac{4}{3}))$
 $(5, 2.214)$

$\theta = \arctan\left(\frac{4}{3}\right)$

VI. Convert the rectangular equation to polar form. solve for r

22. $x^2 + y^2 = 9$
 $r^2 = 9$
 $r = 3$

23. $y = 4$
 $r \sin \theta = 4$
 $r = \frac{4}{\sin \theta}$

24. $y = x$
 $r \sin \theta = r \cos \theta$
 $\sin \theta = \cos \theta$

VII. Convert each polar equation to rectangular form.

25. $r = -5 \sec \theta$
 $r = \frac{-5}{\cos \theta}$
 $r \cos \theta = -5$
 $x = -5$

26. $(r = 4 \sin \theta) r$
 $r^2 = 4r \sin \theta$
 $x^2 + y^2 = 4y$
 $x^2 + y^2 - 4y = 0$
 $x^2 + (y-2)^2 = 4$

26. $r = 4$ $r^2 = 16$
 $x^2 + y^2 = 16$

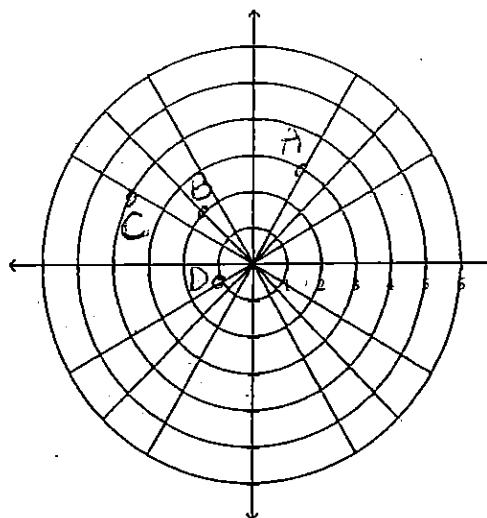
I. Plot and Label the following points with the polar coordinates given (use the letter to label the point).

1. A $(3, \frac{\pi}{3})$

2. B $(-2, \frac{-\pi}{4})$

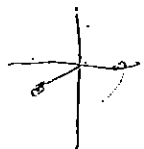
3. C $(4, \frac{-7\pi}{6})$

4. D $(-1, \frac{\pi}{6})$



II. Name the following in 2 other ways each (use only one rotation).

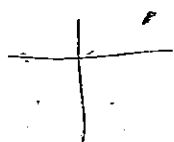
5. $(2, \frac{-5\pi}{6})$



5. $(-2, \frac{\pi}{6})$

$(2, \frac{7\pi}{6})$ $(2, \frac{-11\pi}{6})$

6. $(-2, \frac{4\pi}{3})$

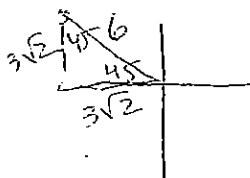


6. $(2, \frac{\pi}{3})$

$(2, \frac{-5\pi}{3})$ $(-2, \frac{-2\pi}{3})$

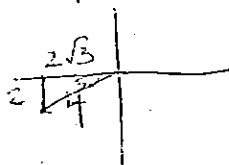
III. Transform the given polar coordinates to rectangular ordered pairs.

7. $(6, \frac{3\pi}{4})$ to (x, y)



7. $(-3\sqrt{2}, 3\sqrt{2})$

8. $(-4, \frac{\pi}{6})$ to (x, y)



8. $(-2\sqrt{3}, -2)$

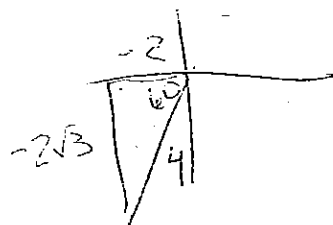
IV. Transform the given rectangular coordinates to polar coordinate ordered pairs.

9. $(-3, -3)$ to (r, θ)



9. $(3\sqrt{2}, \frac{5\pi}{4})$
OR $(-3\sqrt{2}, \frac{\pi}{4})$

10. $(-2, -2\sqrt{3})$ to (r, θ)



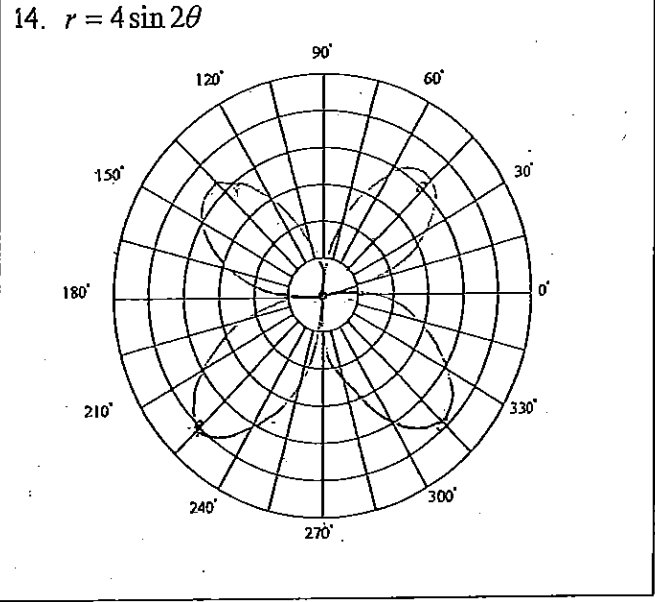
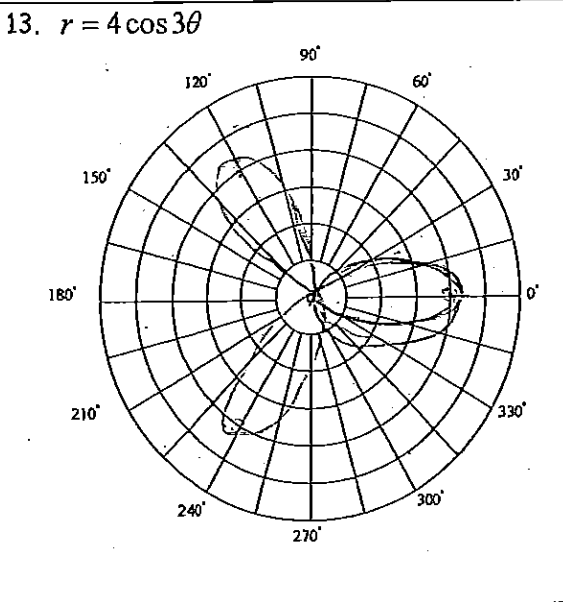
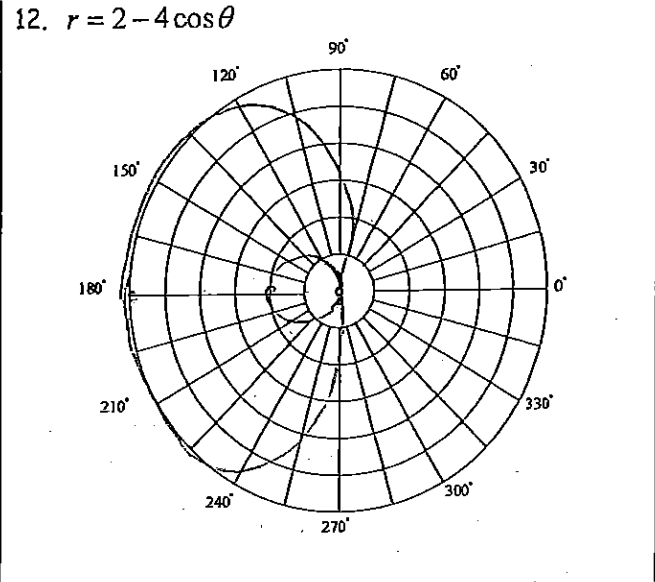
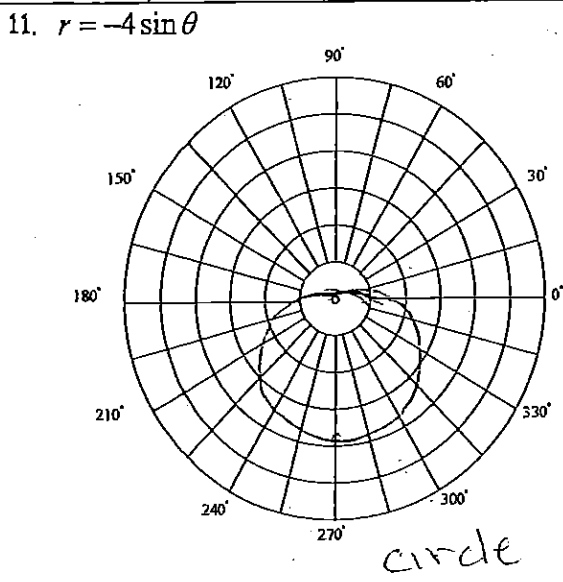
10. $(-4, \frac{\pi}{3})$
OR $4, (\frac{4\pi}{3})$

- Graphs of Polar Equations

I. Match the equation with the correct description. DO WITHOUT GRAPHING!

1. $r = 2 - 2\sin\theta$	B (cardioid)	6. $r = 6\sin 2\theta$	C (rose)	A. Circle B. Cardioid C. Rose D. Limacon with Loop E. Limacon without Loop
2. $r = 3\cos 4\theta$	C (Rose)	7. $r = 4 - 2\sin\theta$	E (no loop)	
3. $r = 9\cos\theta$	A (circle)	8. $r = 3 + 3\sin\theta$	B (cardioid)	
4. $r = 4 - 2\cos\theta$	E (no loop)	9. $r = 5 - 5\cos\theta$	B (cardioid)	
5. $r = 3 + 5\sin\theta$	D (loop)	10. $r = 2 + 3\sin\theta$	D (loop)	

II. Graph each of the following polar equations.



1. Transform the given equation from rectangular form to polar form.

a. $y = 2x^2 - 5x$

$$r \sin \theta = 2(r \cos \theta)^2 - 5r \cos \theta$$

$$\sin \theta = 2r(\cos \theta)^2 - 5 \cos \theta$$

$$\frac{\sin \theta + 5 \cos \theta}{2(\cos \theta)^2} = r$$

c. $x = 3y^2 + 2y$

$$r \cos \theta = 3(r \sin \theta)^2 + 2r \sin \theta$$

$$\cos \theta = 3r(\sin \theta)^2 + 2 \sin \theta$$

$$\frac{\cos \theta - 2 \sin \theta}{3(\sin^2 \theta)} = r$$

b. $y = 2x - 1$

$$r \sin \theta = 2r \cos \theta - 1$$

$$r \sin \theta - 2r \cos \theta = -1$$

$$r(\sin \theta - 2 \cos \theta) = -1$$

$$r = \frac{-1}{\sin \theta - 2 \cos \theta}$$

d. $4x + y = 5$

$$4r \cos \theta + r \sin \theta = 5$$

$$r(4 \cos \theta + \sin \theta) = 5$$

$$r = \frac{5}{4 \cos \theta + \sin \theta}$$

2. Transform the given equation from polar form to rectangular form.

a. $(r = -2 \sin \theta) \cdot r$

$$r^2 = -2r \sin \theta$$

$$x^2 + y^2 = -2y$$

$$x^2 + y^2 + 2y = 0$$

$$x^2 + (y^2 + 2y + 1) = 1$$

$$x^2 + (y+1)^2 = 1$$

Center (0, -1)

$$r = 1$$

b. $(r - 4 \cos \theta = 0) \cdot r$

$$r^2 = 4r \cos \theta$$

$$x^2 + y^2 = 4x$$

$$(x^2 - 4x + 4) + y^2 = 4$$

$$(x-2)^2 + y^2 = 4$$

Center (2, 0)

$$r = 2$$

c. $r = \frac{7}{3 \sin \theta - 2 \cos \theta}$

$$3r \sin \theta - 2r \cos \theta = 7$$

$$3y - 2x = 7$$

$$3y = 2x + 7$$

$$y = \frac{2}{3}x + \frac{7}{3}$$

d. $\frac{r \tan \theta}{r} = \frac{3r^2 \cos \theta}{r}$

$$\tan \theta = 3r \cos \theta$$

$$\frac{y}{x} = 3x$$

$$y = 3x^2$$

3. Simplify.

a. $\frac{\sqrt{-8}\sqrt{-64}}{\sqrt{32}} = \frac{\sqrt{512}}{\sqrt{32}} = \sqrt{16} = 4$

c. $\frac{5}{4-3i} \cdot \frac{4+3i}{4+3i} = \frac{20+15i}{16-9i^2} = \frac{20+15i}{25} = \frac{4+3i}{5}$

b. $\frac{2-\sqrt{4}}{2+\sqrt{4}} = \frac{(2-2i)(2-2i)}{(2+2i)(2-2i)} = \frac{4-8i+4i^2-8i}{4-4i^2} = \frac{-8i-4}{8} = -i$

d. $\frac{-3+5i}{15i} \cdot \frac{i}{i} = \frac{-3i+5i^2}{15i^2} = \frac{-3i-5}{-15}$

Section 10.2 – Polar Equations and Graphs

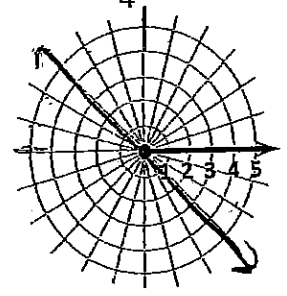
In this section we will learn how to graph various polar equations in the polar coordinate system. The main "shapes" that we will be graphing are circles, lines, cardioids, limaçons, roses, and lemniscates.

LINES

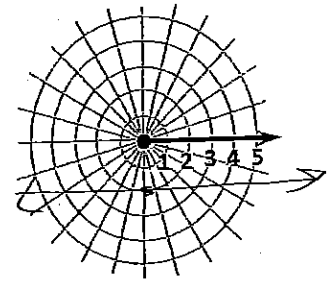
- If you are given a polar equation such as $\theta = \alpha$, then you merely draw a line through the pole that makes an angle of α with the polar axis.
- If you have a polar equation of the form $r \cos \theta = a$, then recall that $x = r \cos \theta$, so in rectangular coordinates that would become $x = a$, which is the equation of a vertical line.
- For a polar equation of the form $r \sin \theta = b$, recall that $y = r \sin \theta$, so that becomes the equation $y = b$, which is the equation of a horizontal line.

Example: Graph the equation.

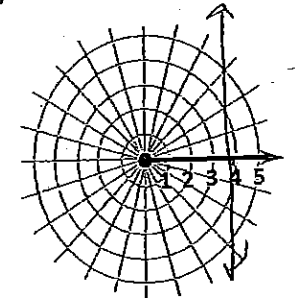
a) $\theta = -\frac{\pi}{4}$



b) $y = -2$
 $r \sin \theta = -2$

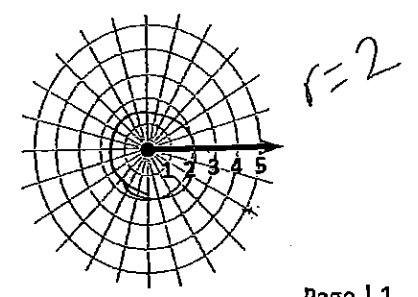
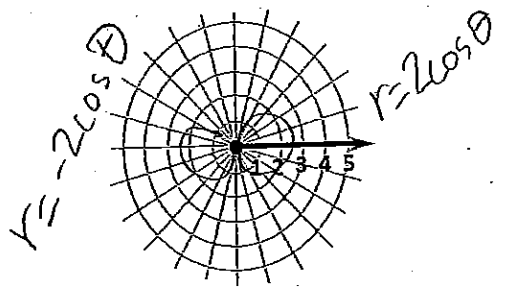
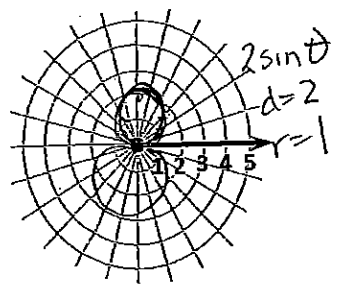


c) $x = 4$
 $r \cos \theta = 4$



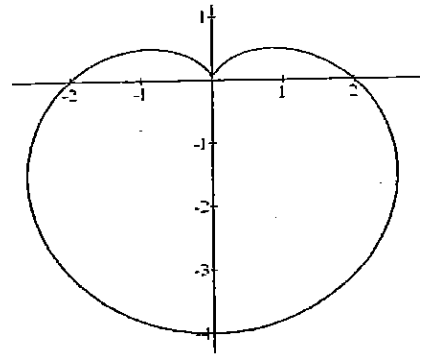
CIRCLES

- Circles of the form $r = a$, where a is a positive number (notice there is no θ in the equation!), form circles centered at the pole with a radius of a . Example: Graph $r = 4$.
- Circles of the form $r = \pm 2a \cos \theta$ have their centers on the x-axis and have a radius of a . If the equation is positive, then the circle is on the right side of the pole. If the equation is negative, then the circle is on the left side of the pole. These circles pass through the pole. Example: Graph $r = -4 \cos \theta$.
- Circles of the form $r = \pm 2a \sin \theta$ have their centers on the y-axis and also have a radius of a . If the equation is positive, then the circle is above the pole. If the equation is negative, then the circle is below the pole. These circles pass through the pole. Example: Graph $r = 2 \sin \theta$.



CARDIOIDS

Cardioids, such as the one shown in the figure on the right, got their name because the shape resembles a heart. Cardioids pass through the pole, and the distance from the pole to the farthest point on the main axis is equal to $2a$. Next to the box below, label where the main portion of the cardioid will appear (above, below, to the left, or to the right of the pole).



Cardioids are characterized by equations of the form

Left	$r = a(1 + \cos \theta)$	$r = a(1 + \sin \theta)$	above
right	$r = a(1 - \cos \theta)$	$r = a(1 - \sin \theta)$	below

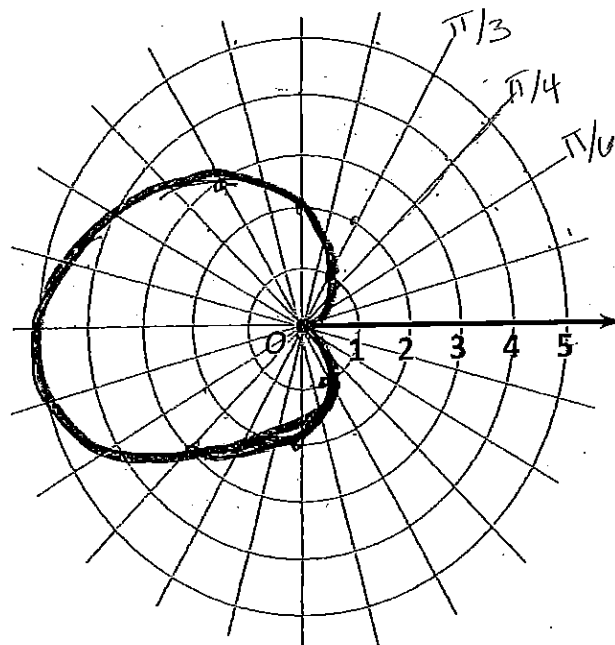
where $a > 0$. The graph of a cardioid passes through the pole.

Once you have established the general shape and orientation of a cardioid, you must plot points on the graph by choosing various values of θ and finding out what r values they result in.

Example: Graph the cardioid $r = 2 - 2\cos\theta$.

First, we note that the cardioid will be centered on the X AXIS (because of the $\cos \theta$) and the main portion will be to the left of the pole (because of the negative sign). We also need to factor out the 2, resulting in the equation $2(1 - \cos \theta)$. We can see that $a = 2$, and thus the farthest point on the main axis will be a distance of 4 away from the pole. At this point we know the general shape of the cardioid. Now we need to find specific points on the graph. The easiest way to do this is to change your graphing calculator MODE from Function to POLAR. Click on Y= then insert the equation. Now go to TblSet and have the TblStart be 0 and the ΔTbl be $\pi/6$. Then click on TABLE to see the r -values that correspond to each value of θ .

θ	$r = 2 - 2\cos\theta$
0	0
$\frac{\pi}{6}$	2.7
$\frac{\pi}{3}$	1
$\frac{\pi}{2}$	2
$\frac{2\pi}{3}$	3
$\frac{5\pi}{6}$	3.7
π	4



LIMAÇONS

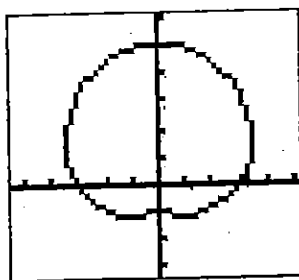
There are 2 types of limaçons: those *with* an inner loop and those *without* an inner loop.

Limaçons without an inner loop are characterized by equations of the form

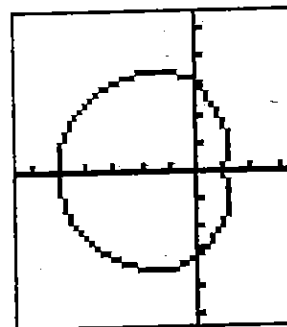
$$r = a + b \cos \theta \quad r = a + b \sin \theta$$

$$r = a - b \cos \theta \quad r = a - b \sin \theta$$

where $a > 0$, $b > 0$, and $a > b$. The graph of a limaçon without an inner loop does not pass through the pole.



$$r = 3 + 2 \sin \theta$$



$$r = 3 - 2 \cos \theta$$

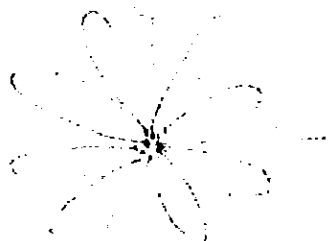
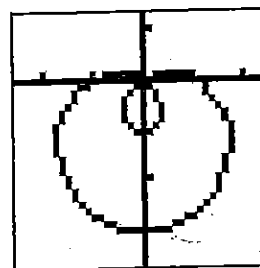
Notice in the box above the specification that $a > b$ (the constant is bigger than the coefficient). This is precisely what makes this limaçon *not* have an inner loop. For limaçons that *do* have an inner loop, $a < b$ (so the constant is smaller than the coefficient). For instance, in the equation $r = 3 + 2 \cos \theta$, $a = 3$ and $b = 2$. Since $a > b$, this limaçon does NOT have an inner loop. But for the equation $r = 2 + 3 \cos \theta$, $a = 2$ and $b = 3$, so since $a < b$, this limaçon DOES have an inner loop. What about the case where $a = b$, such as $r = 2 + 2 \cos \theta$? We can rewrite the equation as $2(1 + \cos \theta)$ and we realize that it is actually a cardioid.

Limaçons follow the same rules as cardioids in terms of where the main part (or bulk) of the graph will appear. For $+\cos \theta$ it will be to the right of the pole. For $-\cos \theta$ it will be to the left of the pole. For $+\sin \theta$ it will be above the pole, and for $-\sin \theta$ it will be below the pole.

For limaçons that do not have an inner loop, the furthest point from the pole is $a + b$ and the closest point to the pole is $a - b$. So assuming each tick mark represents 1 unit in the graphing-calculator-produced graphs at the top of the page, determine the equation of each limaçon.

Although the graphs of limaçons *without* an inner loop do NOT pass through the pole, the graphs of limaçons *with* an inner loop pass through the pole twice. In the graph on the right, pick any starting point and trace the limaçon to see how this happens.

For limaçons that have an inner loop, the length of the inner loop is $b - a$ from the pole and the length of the outer loop is $b + a$ from the pole. What is the equation of this limaçon? $r =$ $1 - 2 \sin \theta$



ROSE CURVES

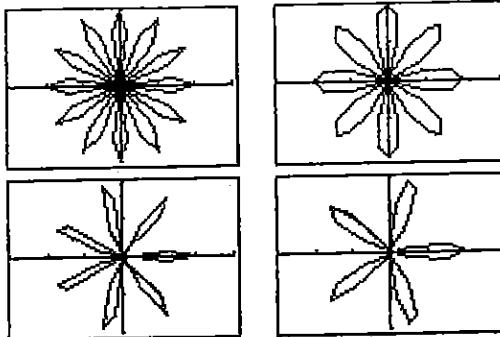
Rose curves are characterized by equations of the form

$$r = a \cos(n\theta), \quad r = a \sin(n\theta), \quad a \neq 0$$

and have graphs that are rose shaped.

If $n \neq 0$ is even, the rose has $2n$ petals:

if $n \neq \pm 1$ is odd, the rose has n petals.



Notice in the equations above that θ has a coefficient. If θ does not have a coefficient (so the equation looks like $r = 2\cos\theta$, for instance), the graph is *not* a rose curve. We have actually already learned how to graph this kind of equation. What will the graph of $r = 2\cos\theta$ be? circle $r=1$

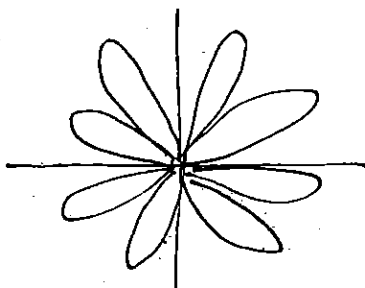
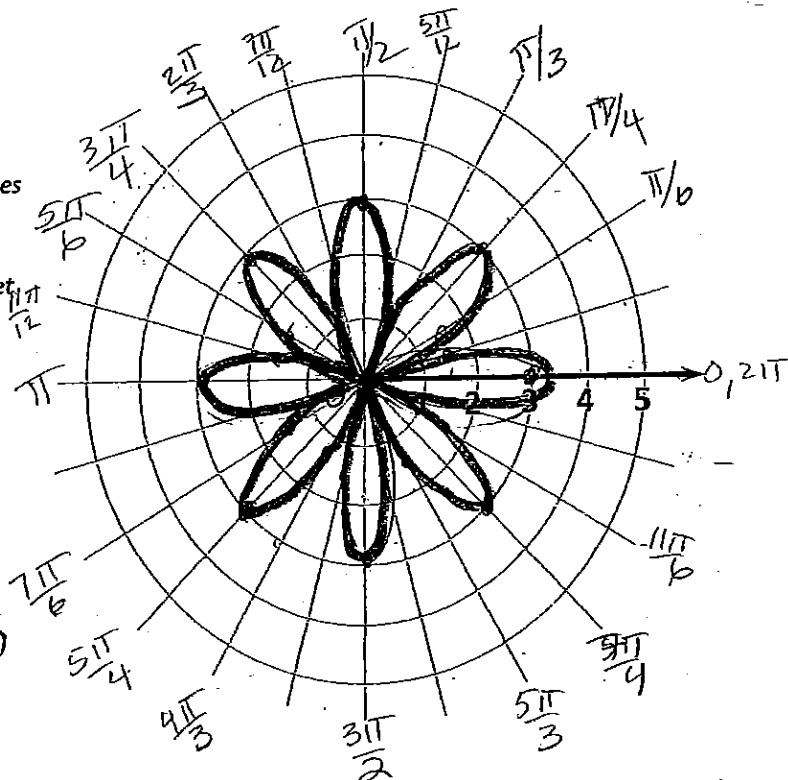
So we realize that in order to make a rose curve, θ must have a coefficient, n . If n is *even*, then the rose will have $2n$ petals. If n is *odd*, then the rose will have n petals. Additionally, if n is *odd* and the equation contains cosine, then one of the petals will lie directly on the x-axis, and if n is *odd* and the equation contains sine, then one of the petals will lie directly on the y-axis. Lastly, the length of each petal will be equal to a (the coefficient).

Example: Graph the polar equation $r = 3\cos(4\theta)$.

First, we identify this as a rose curve. We see that one petal will lie directly on the x-axis and the length of each petal will be 3. Since n is even, we know there will be 8 petals.

θ	$r = 3\cos(4\theta)$
0	-3
$\frac{\pi}{12}$	1.5
$\frac{\pi}{6}$	-1.5
$\frac{\pi}{4}$	-3
$\frac{\pi}{3}$	-1.5
$\frac{5\pi}{12}$	1.5
$\frac{\pi}{2}$	3
$\frac{7\pi}{12}$	1.5
$\frac{2\pi}{3}$	-1.5
$\frac{3\pi}{4}$	-3
$\frac{5\pi}{6}$	-1.5
$\frac{11\pi}{12}$	1.5
π	3

To get all of these values from your calculator's "Table", you need to set ΔTbl to $\pi/12$.



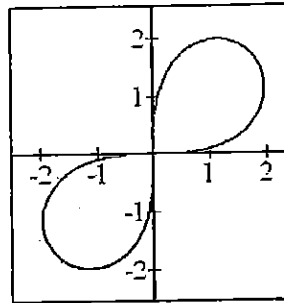
LEMNISCATES

Lemniscates are propeller-shaped graphs, as shown in the figures on the right.

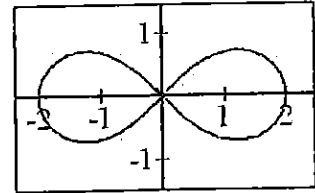
Lemniscates are characterized by equations of the form

$$r^2 = a^2 \sin(2\theta) \quad r^2 = a^2 \cos(2\theta)$$

where $a \neq 0$, and have graphs that are propeller shaped.



$$r^2 = 6 \sin(2\theta)$$



$$r^2 = 4 \cos(2\theta)$$

Notice that the r is SQUARED in these equations. In order to put them into your calculators, you need to take the square root of both sides and put the square root into your calculator. For instance, to get the graph of $r^2 = 6 \sin(2\theta)$, I typed

$r = \sqrt{6 \sin(2\theta)}$ into my calculator. To graph the equation on the right, what would you put into your calculator?

$r = \sqrt{4 \cos(2\theta)}$ Lastly, notice that the equations containing *sine* create lemniscates in Quadrants 1 and 3, whereas the ones containing *cosine* create lemniscates along the x-axis.

Example: Identify the type of graph and the key features of each graph. DO NOT GRAPH.

a) $r = 2 + 4 \cos \theta$ inner loop
right of pole
inner loop $\theta = (2, \pi)$
outer loop $\theta = (6\pi)$ *imagm*

c) $r = 3 - \sin \theta$ *imagm*
no inner loop
 $(0, 2)$ $(0, -4)$
under the pole $(0, 0)$

e) $r = 6 \cos \theta$ Circle
 $r = 3$
right side of pole
diameter on x axis

b) $r = 2 \sin(5\theta)$ rose

5 petals

$l = 2$

single petal on positive
of axis

d) $r^2 = 9 \sin(2\theta)$ lemniscate
petals are in 1st + 3rd
quadrant on a
diagonal (45° angle)

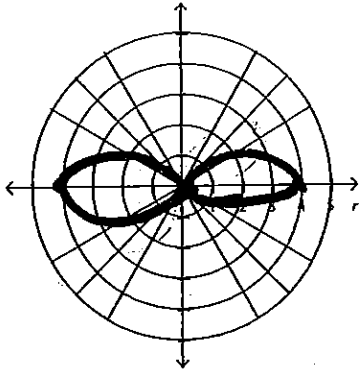
f) $r \cos \theta = -2$ vertical
line

at $x = -2$

Sketch the graphs of the following polar equations.

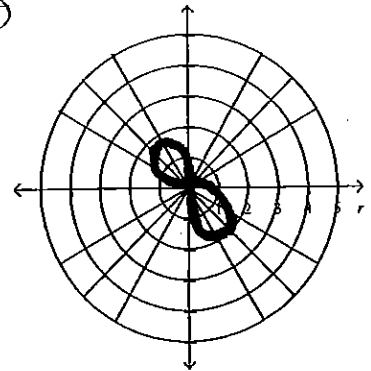
1. $r^2 = 16 \cos 2\theta$

$\pm \sqrt{16 \cos 2\theta}$



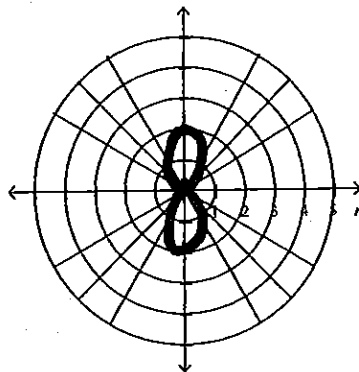
2. $r^2 = -9 \sin 2\theta$

$\pm \sqrt{-9 \sin 2\theta}$



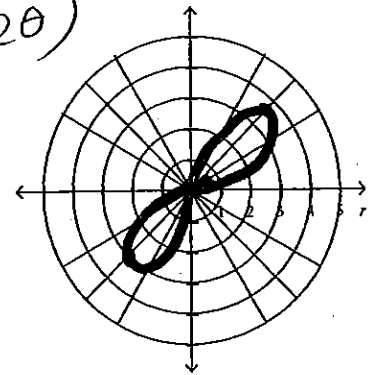
3. $r^2 = -4 \cos 2\theta$

$\pm \sqrt{-4 \cos 2\theta}$

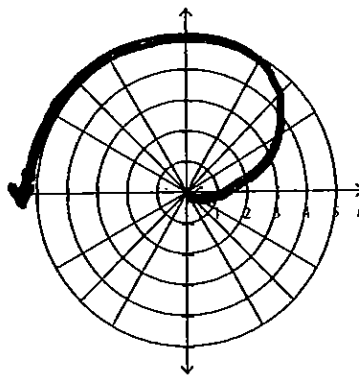


4. $r^2 = 25 \sin 2\theta$

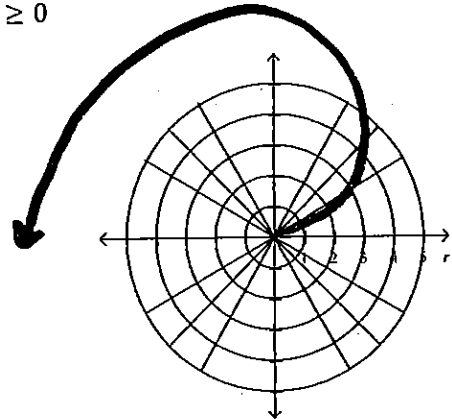
$\pm \sqrt{25 \sin 2\theta}$



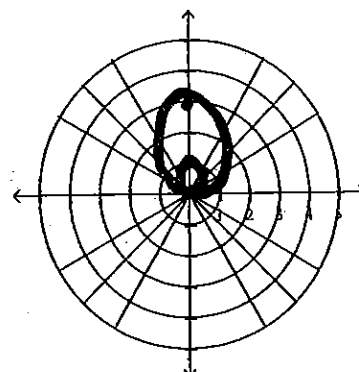
5. $r = \theta, \theta \geq 0$



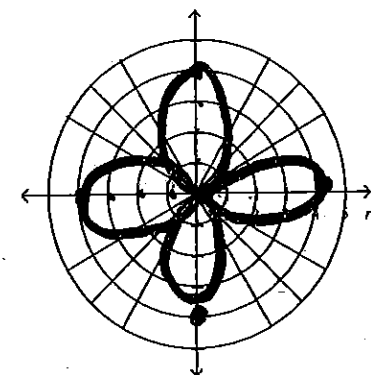
6. $r = 2\theta, \theta \geq 0$



7. $r = 1 + 2 \sin \theta$



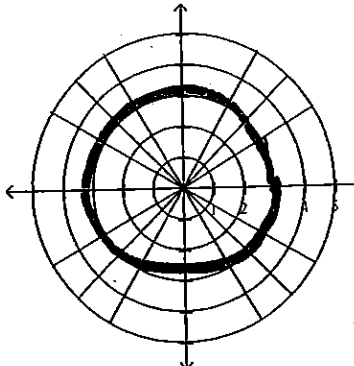
8. $r = -4 \cos 2\theta$



Pre-Calculus Unit 6 Homework

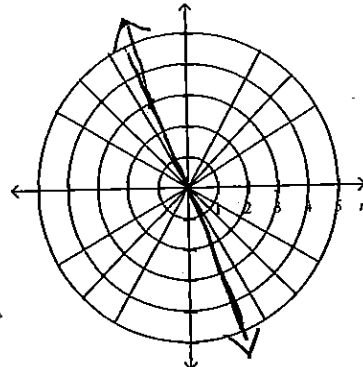
Name Key 2015

9. $r = 3$



Circle
 $r=3$
 $(0,0)$ = center

11. $\theta = 2$

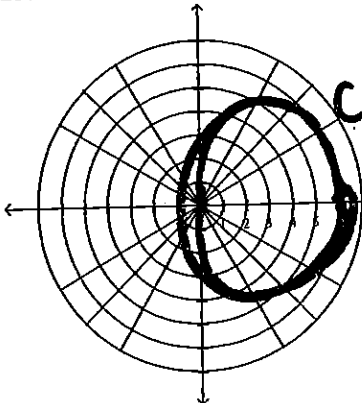


2 radians

$2 \cdot \frac{180}{\pi} = 114.6^\circ$

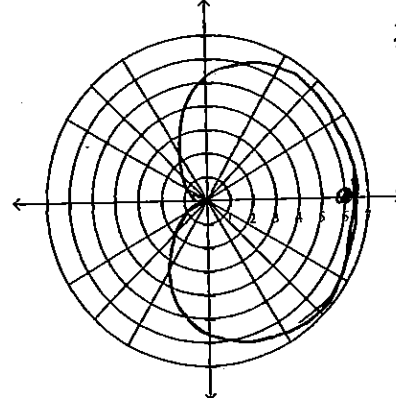
line

13. $r = 6 \cos \theta$



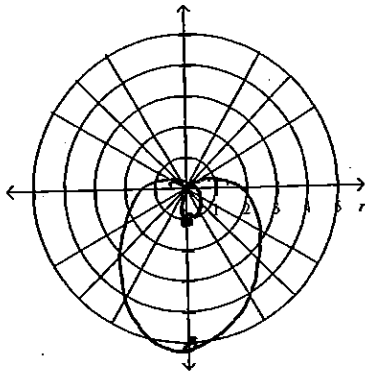
Circle
 $r=3$
 $(0,0)$
 $(6,0)$

15. $r = 3 + 3 \cos \theta$



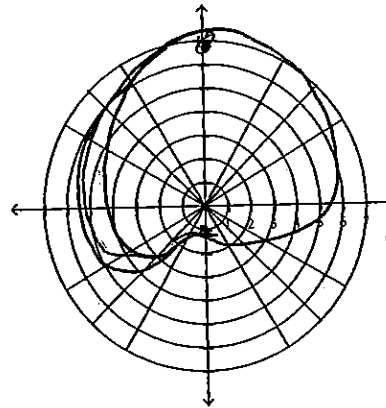
$3+3=6$ $(6,0)$
 $3-3=0$ $(0,0)$

17. $r = 2 - 3 \sin \theta$



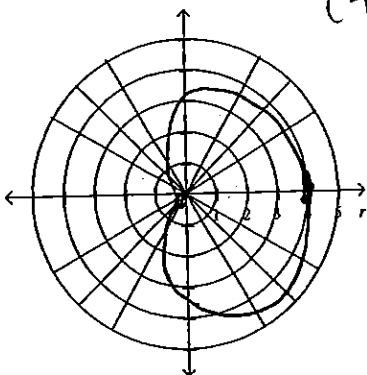
$2-3=-1$
 $(0,-1)$
 $2+3=5$
 $(0,5)$

19. $r = 4 + 3 \sin \theta$



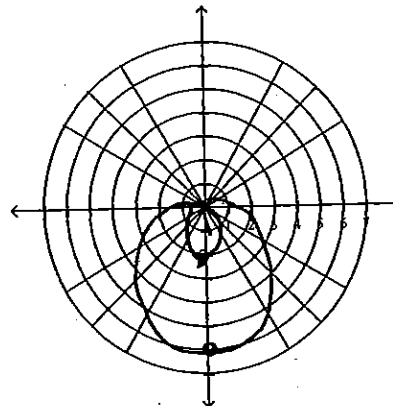
$4+3=7$
 $4-3=1$
 $3-4=-1$
 $(0,7)$
 $(0,-1)$

21. $r = -2 + 2 \cos \theta$



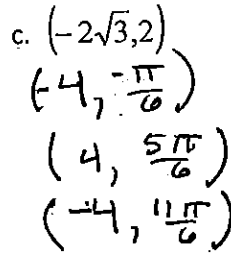
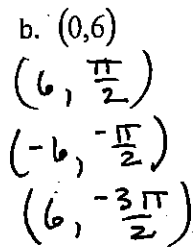
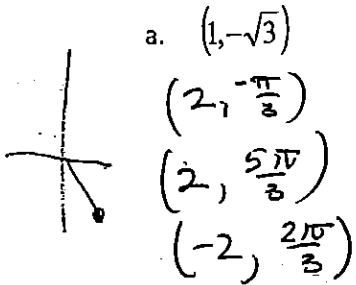
$(0,0)$
 $(4,0)$

23. $r = 2 - 4 \sin \theta$

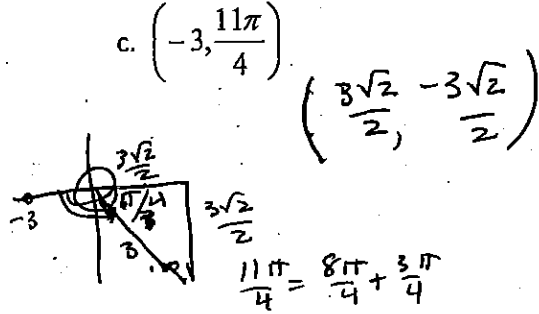
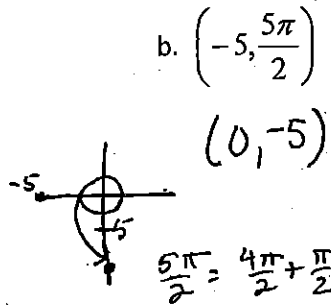
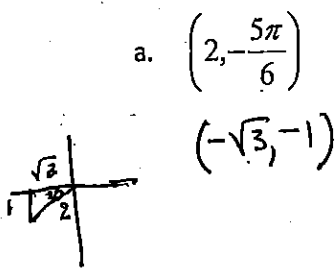


$2-4=-2$ $(0,-2)$
 $2+4=6$ $(0,6)$

1. Give at least three polar coordinates for each rectangular coordinate. (these aren't the only answers)



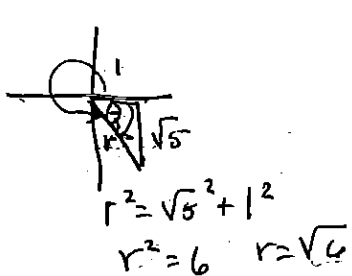
2. Given the polar coordinate, find the rectangular coordinates for each point.



3. Find the $|2 - 5i|$

$$= \sqrt{2^2 + 5^2} = \sqrt{29}$$

4. Write the given complex number in polar form. $1 - i\sqrt{5}$



$$\theta = \tan^{-1}\left(\frac{\sqrt{5}}{1}\right) = 1.15 \text{ radians}$$

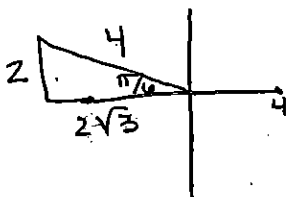
$$(\sqrt{6}, 1.15)$$

$$= \sqrt{6} (\cos(1.15) + i \sin(1.15))$$

or

$$\sqrt{6} (\cos(5.13) + i \sin(5.13))$$

5. Write the given complex number in rectangular form. $4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

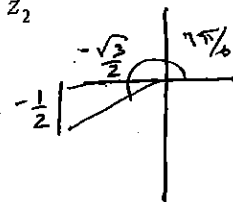


$$-2\sqrt{3} + 2i$$

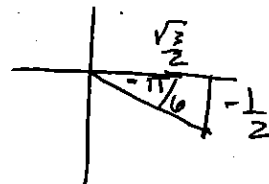
$$4\left(-\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right)$$

6. Given $z_1 = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ and $z_2 = 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ find $z_1 z_2$ and $\frac{z_1}{z_2}$

$$\begin{aligned} z_1 z_2 &= 3 \cdot 4 \left(\cos\left(\frac{\pi}{2} + \frac{2\pi}{3}\right) + i \sin\left(\frac{\pi}{2} + \frac{2\pi}{3}\right) \right) \\ &= 12 \left(\cos\frac{7\pi}{6} + i \sin\frac{7\pi}{6} \right) \\ &= 12 \left(-\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2} \right) = \boxed{-6\sqrt{3} + 6i} \end{aligned}$$

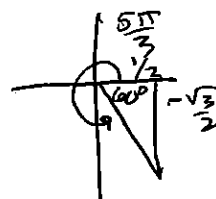


$$\begin{aligned} \frac{z_1}{z_2} &= \frac{3}{4} \left(\cos\left(\frac{\pi}{2} - \frac{2\pi}{3}\right) + i \sin\left(\frac{\pi}{2} - \frac{2\pi}{3}\right) \right) \\ &= \frac{3}{4} \left(\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right) \right) \\ &= \frac{3}{4} \left(\frac{\sqrt{3}}{2} + i \cdot \frac{-1}{2} \right) = \boxed{\frac{3\sqrt{3}}{8} - \frac{3i}{8}} \end{aligned}$$



7. Use DeMoivre's Theorem to find $\left[\sqrt{3}\left(\cos\frac{5\pi}{18} + i\sin\frac{5\pi}{18}\right)\right]^6$

$$\begin{aligned} z^6 &= (\sqrt{3})^6 \left(\cos\left(6 \cdot \frac{5\pi}{18}\right) + i \sin\left(6 \cdot \frac{5\pi}{18}\right) \right) \\ &= 27 \left(\cos\frac{5\pi}{3} + i \sin\frac{5\pi}{3} \right) \\ &= 27 \left(\frac{1}{2} + i \cdot \frac{-\sqrt{3}}{2} \right) \\ &= \boxed{\frac{27}{2} - \frac{27i\sqrt{3}}{2}} \end{aligned}$$



8. Convert each polar equation to a rectangular equation.

a. $(r = 6\cos\theta + 5\sin\theta) \cdot r$

$$r^2 = 6r\cos\theta + 5r\sin\theta$$

$$x^2 + y^2 = 6x + 5y$$

$$x^2 - 6x + 9 + y^2 - 5y + 6.25 = 9 + 6.25$$

$$\boxed{(x-3)^2 + (y-2.5)^2 = 15.25}$$

b. $r^2 \sin 2\theta = 2$

$$r^2 \cdot 2\sin\theta \cos\theta = 2$$

$$2r\sin\theta r\cos\theta = 2$$

$$2xy = 2$$

$$yx = 1$$

$$\boxed{y = \frac{1}{x}}$$

9. Convert each rectangular equation to a polar equation.

a. $x^2 + y^2 = 6x$

$$\frac{r^2 = 6r\cos\theta}{r}$$

$$\boxed{r = 6\cos\theta}$$

b. $2x + 3y = 8$

$$2r\cos\theta + 3r\sin\theta = 8$$

$$r(2\cos\theta + 3\sin\theta) = 8$$

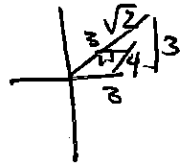
$$\boxed{r = \frac{8}{2\cos\theta + 3\sin\theta}}$$

PCH Ch 11 Review (solutions)

①

$$1) m \pm 3\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$m = 3 + 3i$$



$$mr = 3\sqrt{2} \cdot 5 \left(\cos \left(\frac{\pi}{4} + \frac{3\pi}{4} \right) + i \sin \left(\frac{\pi}{4} + \frac{3\pi}{4} \right) \right)$$

$$= 15\sqrt{2} (\cos \pi + i \sin \pi)$$

$$= 15\sqrt{2} (-1 + i \cdot 0)$$

$$= \boxed{-15\sqrt{2}}$$

$$2) \textcircled{a} y = 4x^2 + 3x$$

$$r \sin \theta = \frac{4r^2 \cos^2 \theta + 3r \cos \theta}{r}$$

$$\sin \theta = 4r \cos^2 \theta + 3 \cos \theta$$

$$\frac{\sin \theta - 3 \cos \theta}{\cos^2 \theta} = \frac{4r \cos^2 \theta}{4 \cos^2 \theta}$$

$$r = \frac{\sin \theta - 3 \cos \theta}{4 \cos^2 \theta}$$

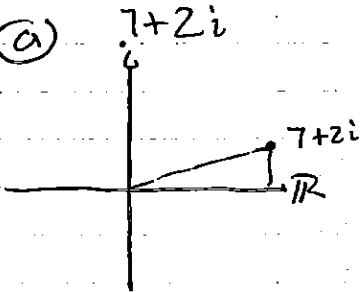
$$\textcircled{b} 8x + 3y = 9$$

$$8r \cos \theta + 3r \sin \theta = 9$$

$$r(8 \cos \theta + 3 \sin \theta) = 9$$

$$r = \frac{9}{8 \cos \theta + 3 \sin \theta}$$

$$3) \textcircled{a} 7 + 2i$$

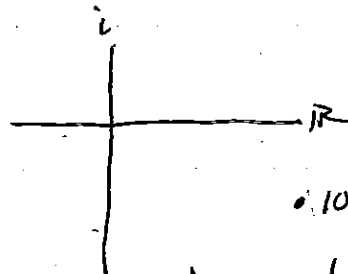


$$|7 + 2i| =$$

$$= \sqrt{7^2 + 2^2}$$

$$= \sqrt{53}$$

$$\textcircled{b}$$



$$|10 - 5i| =$$

$$= \sqrt{10^2 + 5^2} = \sqrt{125}$$

$$= 5\sqrt{5}$$

$$4) \textcircled{a} \frac{3 - \sqrt{-4}}{2 + i - 16} = \frac{3 - 2i}{2 + 4i} \cdot \frac{2 - 4i}{2 - 4i}$$

$$= \frac{6 - 12i - 4i + 8i^2}{4 - 16i^2}$$

$$= \frac{-2 - 16i}{20} = \boxed{\frac{-1 - 8i}{10}}$$

$$\textcircled{b} (9 - \sqrt{-8})(3 + \sqrt{-2})$$

$$= (9 - 2i\sqrt{2})(3 + i\sqrt{2})$$

$$= 27 + 9i\sqrt{2} - 6i\sqrt{2} - 2i^2(2)$$

$$= 27 + 3i\sqrt{2} + 4 = \boxed{31 + 3i\sqrt{2}}$$

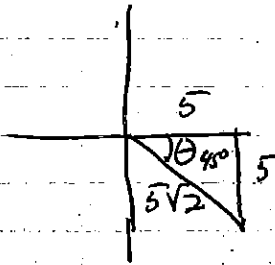
$$5) r = -7 \csc \theta$$

$$r = \frac{-7}{\sin \theta}$$

$$r \sin \theta = -7$$

$$\boxed{u = -7}$$

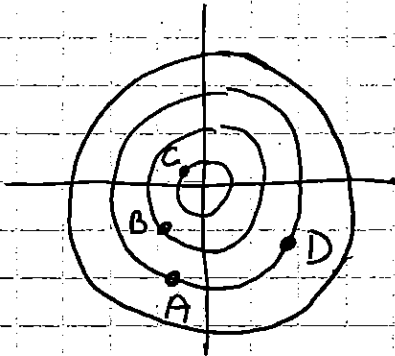
7) $5-5i \rightarrow$ Polar



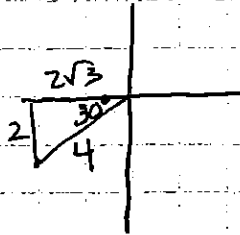
$$5\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$

$$5\sqrt{2} \left(\cos -\frac{\pi}{4} + i \sin -\frac{\pi}{4} \right)$$

8)



9) $(-2\sqrt{3} - 2i)^5 = 4 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right)^5$



$$z^5 = 4^5 \left(\cos \left(5 \cdot \frac{7\pi}{6} \right) + i \sin \left(5 \cdot \frac{7\pi}{6} \right) \right)$$

$$= 1024 \left(\cos \frac{35\pi}{6} + i \sin \frac{35\pi}{6} \right)$$

$$1024 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right)$$

$$= 1024 \left(\frac{\sqrt{3}}{2} + i \cdot \frac{-1}{2} \right)$$

$$\boxed{512\sqrt{3} - 512i}$$

9) next page

9. Use DeMoivre's Theorem to find the given complex number.

$$(-2\sqrt{3} - 2i)^5$$

10. Sketch the curves for the given polar equation using your graphing calculator. Label important parts of the graph.

a. $r = 5\cos 7\theta$ flower 7 petals

b. $r = 8\sin \theta$ circle $d = 8$

c. $r = 3 + 3\cos \theta$ cardioid

$$d = 3 + 3 = 6$$

